

Grade 6 Math Circles

Winter 2013

Fractions

Review

A *fraction* is how we express parts of a whole.

We call the top part of a fraction the **numerator**, and the bottom part of a fraction the **denominator**. The bar of the fraction is simply called the **fraction bar**.

The numerator is how many parts we have of a whole.

The denominator is how many parts the whole is divided into.

So, when we are discussing fractions we are actually talking about how many parts of a whole we have, in terms of how the whole is divided up.

Adding

$$\frac{1}{2} + \frac{1}{3}$$

Steps:

1. **Make a common denominator.** $\frac{3}{6} + \frac{2}{6}$
2. **Add the numerators.** $\frac{5}{6}$
3. **Reduce the fraction into lowest terms, if possible.** Not possible in this example.

Subtracting

$$\frac{1}{2} - \frac{1}{3}$$

Steps:

1. Make a common denominator. $\frac{3}{6} - \frac{2}{6}$
2. Subtract the numerators. $\frac{1}{6}$
3. Reduce the fraction into lowest terms, if possible. Not possible in this example.

Multiplying

$$\frac{1}{2} \times \frac{1}{3}$$

Steps:

1. Multiply the numerators together. $1 \times 1 = 1$
2. Multiply the denominators together. $2 \times 3 = 6$
3. Put the multiplied numerators over the multiplied denominators. $\frac{1}{6}$
4. Reduce the fraction into lowest terms, if possible. Not possible in this example.

Dividing

$$\frac{1}{2} \div \frac{1}{3}$$

Steps:

1. Take the reciprocal of the second fraction. $\frac{1}{3} \rightarrow \frac{3}{1}$
2. Multiply the fractions together $\frac{1}{2} \times \frac{3}{1} = \frac{3}{2}$
3. Reduce the fraction into lowest terms, if possible. Not possible in this example

Improper and Mixed Fractions

A *proper fraction* is a fraction that has a numerator less than its denominator.

An *improper fraction* is a fraction that has a numerator that is larger than its denominator.

A *mixed fraction* is a proper fraction combined with a whole number.

How do we change mixed fractions into improper fractions?

$$3\frac{1}{2} = \frac{7}{2}$$

Steps:

1. Multiply the whole number by the denominator of the proper fraction.
2. Add the product to the numerator of the proper fraction.
3. Let that sum be the numerator of the mixed fraction
4. The denominator of the proper fraction stays the denominator of our improper fraction.

What about changing improper fractions to mixed fractions?

$$\frac{5}{2} = 2\frac{1}{2}$$

Steps:

1. Divide the numerator by denominator and take the whole number of that solution as the whole number of the mixed fraction.
2. Multiply the denominator of the improper fraction, by the whole number just found.
3. Subtract that product from the numerator of the improper fraction.
4. Let that difference be the numerator of the mixed fraction.

Improper fractions represent the number of parts of more than one whole. For example $\frac{7}{2}$ means we have 7 parts of a certain amount of pizza's that have been divided into 2.

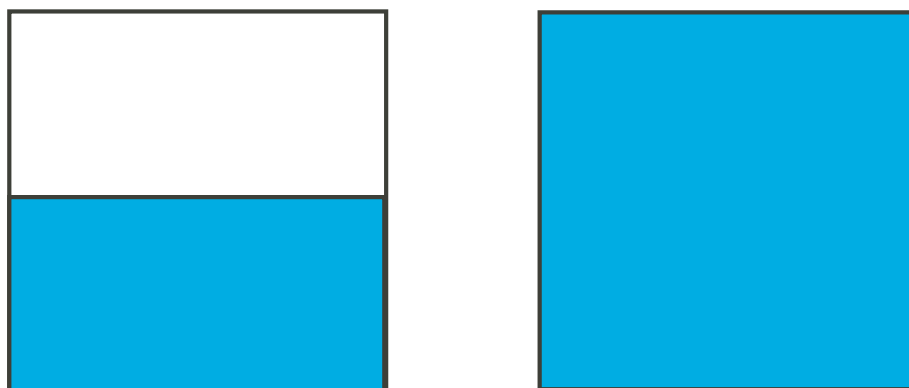
That is the benefit of mixed fractions as opposed to improper fractions. With mixed fractions we know how the exact amount of pizza's. The mixed fraction of the improper fraction $\frac{7}{2}$ is $3\frac{1}{2}$. This means we have 3 whole pizza's and half a pizza.

From this we can figure out that we have 7 parts of 4 pizza's that have been divided into 2.

Fractions and Ratios

A ratio is a comparison between two things.

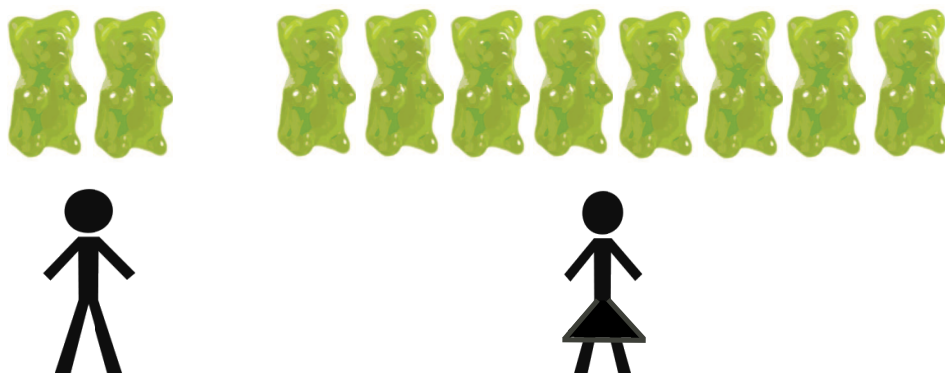
A fraction can also be described as a comparison of two things. Take for example the squares below.



The first square is only half coloured in, but the second is entirely coloured. We can say the first square is $\frac{1}{2}$ of the second square. We are comparing the two squares, in particular, how much of the squares are coloured.

Example 1

Kendra has 8 gummy bears, and Jeff has 2. How many gummy bears does Jeff have compared to Kendra?



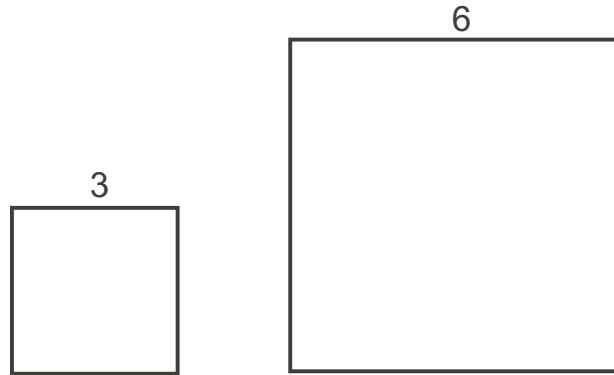
We can say he has $\frac{2}{8} = \frac{1}{4}$ gummy bears.

However, we can also say he has a ratio of 1 : 4 gummy bears compared to Kendra. (Or the ratio of their gummy bears is 1 : 4.) This means for every 8 gummy bears, Jeff has two.

As you can see, ratio are just fractions!

Example 2

The square on the left has side lengths 3 and the square on the right has side lengths 6.



The perimeter of the left square is 12 cm and the perimeter of the right square is 24 cm.

If we compare the squares, the left square is $\frac{12}{24} = \frac{1}{2}$ of the right square.

Meaning the perimeters of the square have a ratio of 1 : 2.

Exercise

Looking at the same squares, the area of the left square is 9 cm² and the area of the right square is 36 cm².

What is ratio between the area of the squares?

The left square is $\frac{9}{36} = \frac{1}{4}$ of the right square.

The ratio between the area of the two squares is 1 : 4.

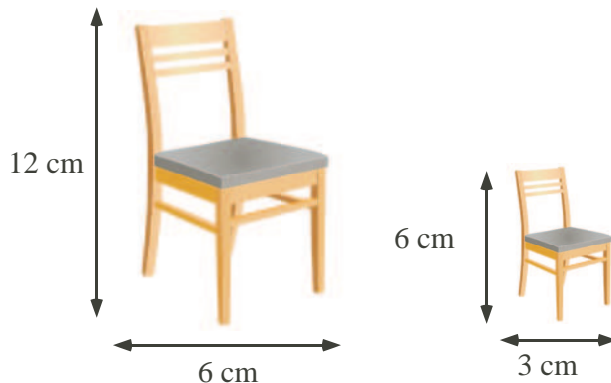
However, since it wasn't specified in the question what we are comparing, we can say the area of right square is $\frac{36}{9} = \frac{4}{1}$ of the area of left square.

So, the ratio between the area of right square compared to the area of left square is 4 : 1.

Scaling

Scaling is a transformation that enlarges or diminished objects, and it is a very handy tool. Scaling is a big part of reading maps. If you can read a map's scale, you can easily identify where you are and how far you really are form your destination. Scaling is also used in scale modeling. When home designers are planning how to decorate rooms they use scale models. And the way a city is laid out properly is also due to scale modeling.

Below is an example of scaling.



The chair on the left has a height of 12, and an overall width of 6. The chair on the right has a height of 6 and an overall width of 3. If we apply the same method we did when comparing the squares, we can say the right chair is half the size of the left chair.

Why? Well if we take the ratio between the heights we have a ratio of $\frac{6}{12} = \frac{1}{2} = 1 : 2$.

The ratio between the widths are $\frac{3}{6} = \frac{1}{2} = 1 : 2$.

Since all the ratios of the chairs are $1 : 2$, or $\frac{1}{2}$, we can say the right chair is half the size of the left chair.

This works both ways, meaning we can say the left chair is double the size of the right chair. What changes is how we take the ratio.

This time we take the ratio of the heights to be $\frac{12}{6} = \frac{2}{1} = 2 : 1$.

And the ratio of the widths to be $\frac{6}{3} = \frac{2}{1} = 2 : 1$.

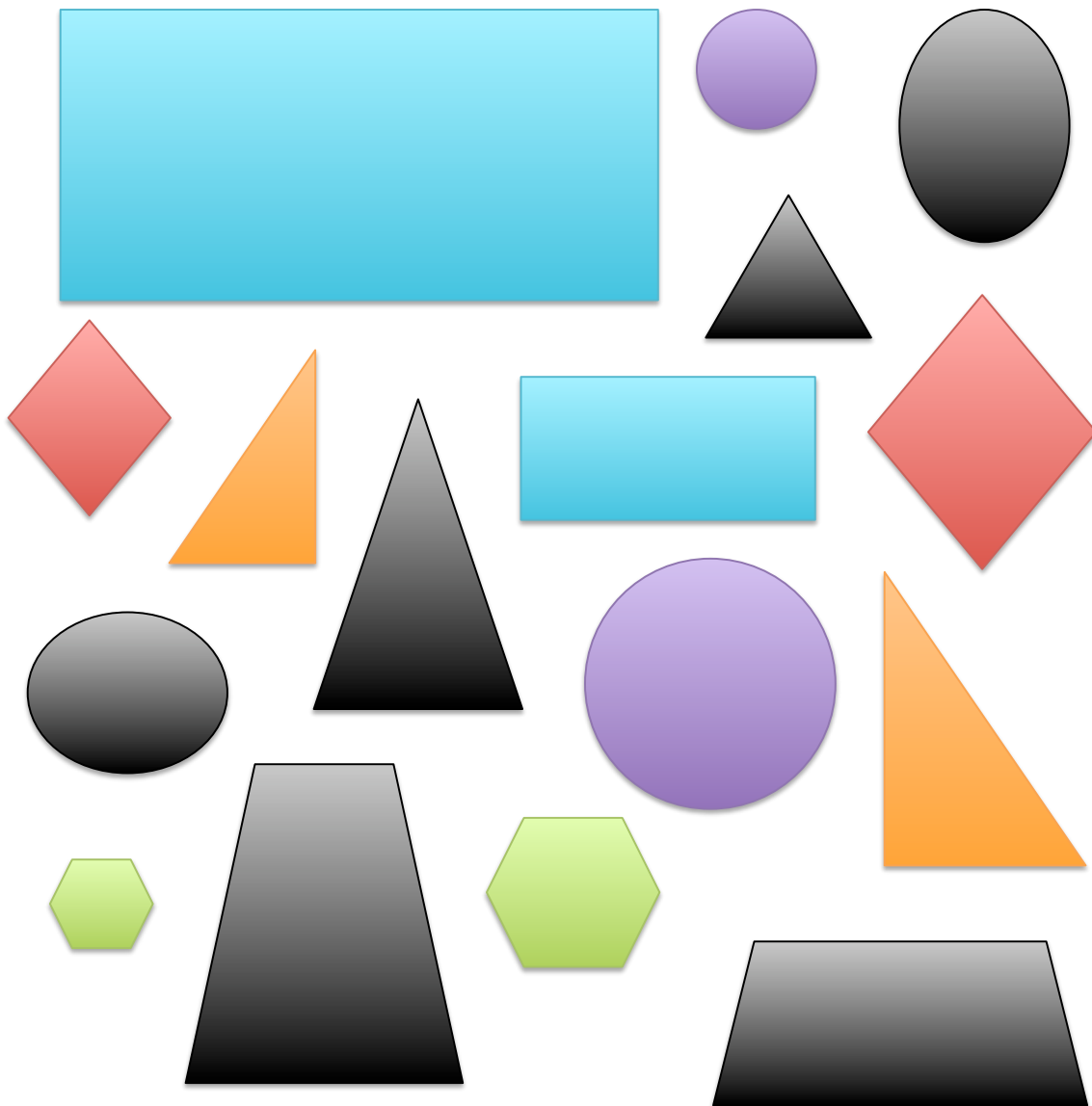
A ratio of $2 : 1$, or the fraction $\frac{2}{1}$, is simply 2, which means the left chair is double the size of the right chair.

We also say that these two chairs are similar, since they have the same ratio between all their dimensions.

Similar Shapes

The same thing can be said about shapes whose dimensions can be related with a ratio. Looking back at our example with squares, we can say the two squares are similar because their side lengths, area and perimeter can be related with ratios.

Out of the following shapes, which ones are similar?

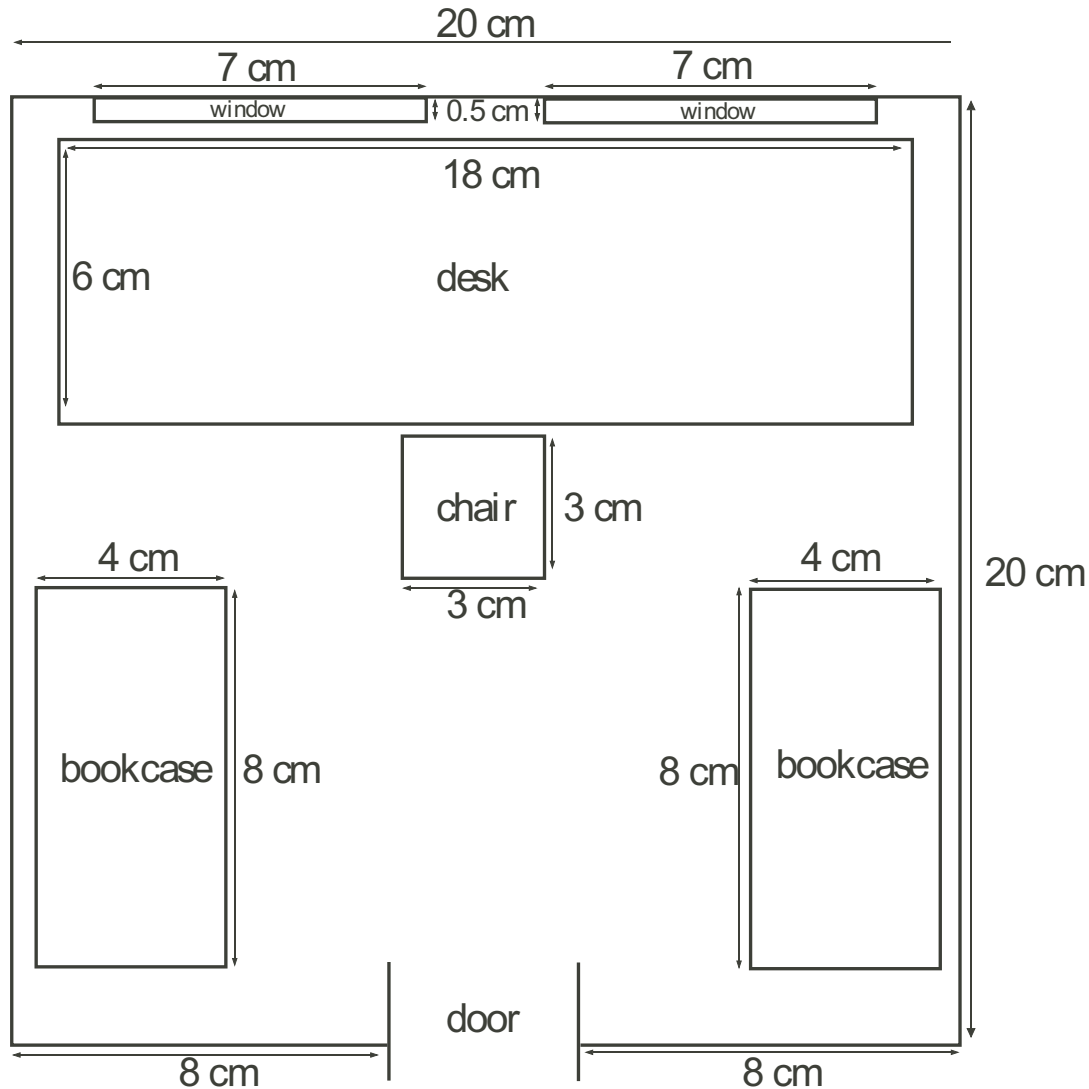


- the blue rectangles
- the red rhombus
- the purple circles
- the orange right angles triangles
- the green hexagons

Scale Models

Let's take a look at the room below.

If all the items, including the room itself, is scaled down by a ratio of 1 : 20, what are the real dimensions of the room and items?



- room 4 m by 4 m
- desk 3.6 m by 1.2 m
- bookcase 0.8 m by 1.6 m
- window 1.4 m by 0.1 m
- chair 0.6 m by 0.6 m

Map Scales

When reading a map there are obviously many things you need to know. What the symbols represent, where is north, but another thing that is important to know is how to read the scale.

Normally maps are represented using what we call a **linear or bar scale**, that indicate by how much the distances have been scaled down. An example is on the next page:



By how much is everything scaled down to in the example above?

1 : 1,000,000. Meaning all distances that you measure in cm are $\frac{1}{1,000,000}$ of the real distance in km.

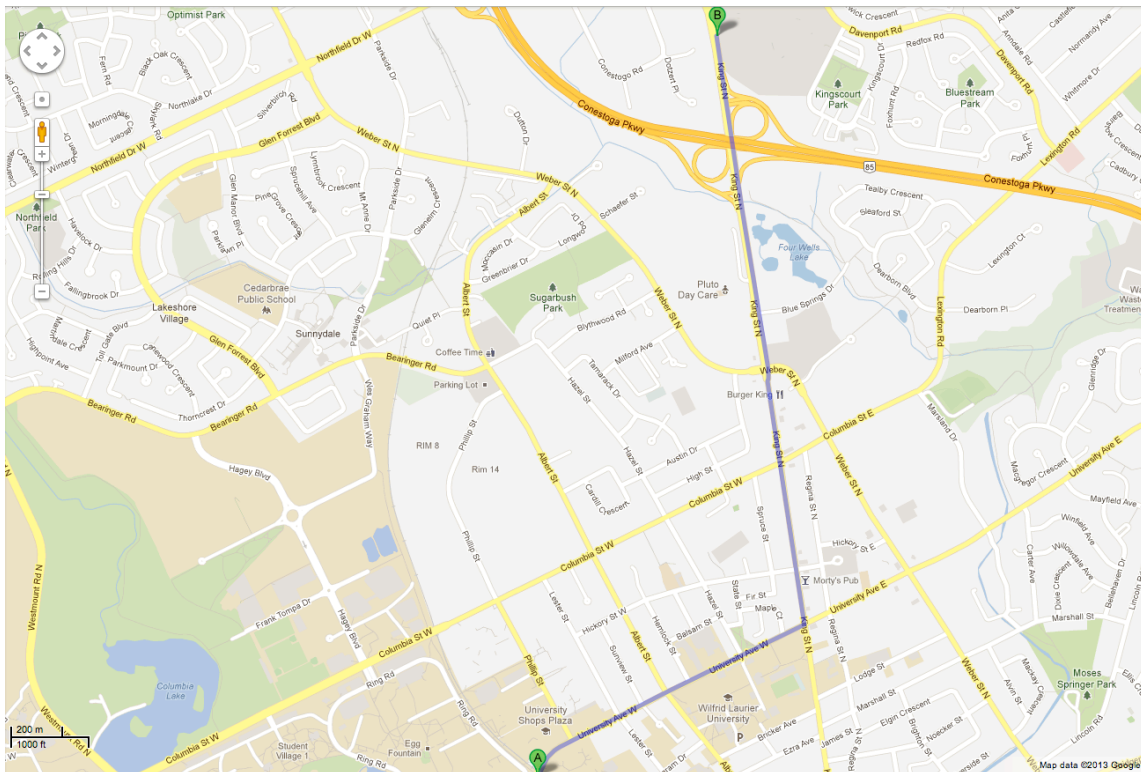
This is what we call a small scale because the distances we are measuring, are really small compared to what they are in real life.

A large scale is any scale with a ratio between 1 : 0 - 1 : 600,000. And example is below.



Here the scale is 1 cm : 1 km. Which is equivalent to the 1 : 100,000,000.

Below is a map representing Waterloo, how far is the University of Waterloo from Conestoga Mall in real life?



$$1 \text{ cm} : 200 \text{ m} = 1 : 20,000; 20,000 \times 12 = 240,000 \text{ cm} = 2400 \text{ m} = 2.4 \text{ km}$$

Equivalent Fractions

Equivalent fractions, or ratios, are fractions or ratios of equal value.

Are the following fractions equivalent?

1. T or F $\frac{5}{10} = \frac{1}{2}$

2. T or F $\frac{1}{24} = \frac{2}{12}$

3. T or F $\frac{25}{50} = \frac{6}{5}$

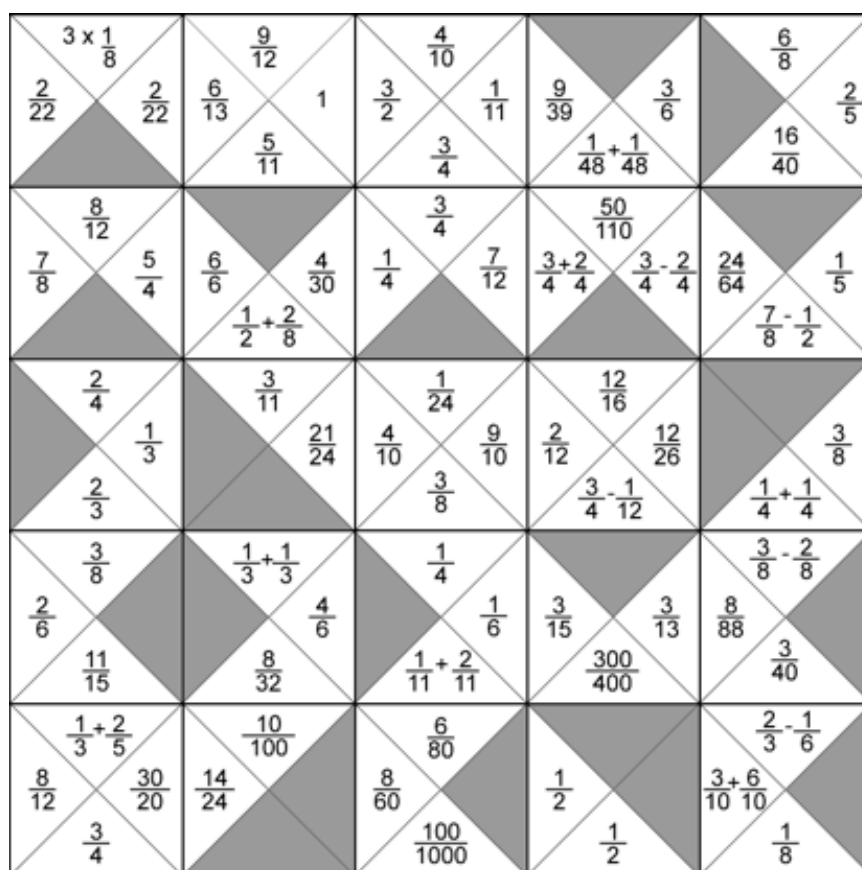
4. T or F $\frac{4}{6} = \frac{20}{30}$

5. T or F $\frac{10}{6} = \frac{30}{36}$

6. T or F $\frac{6}{7} = \frac{18}{21}$

Fractions Jigsaw

Each square is divided into 4 sections, each containing either a fraction, an expression or a blank space. Place the squares so that equivalent values are beside each other. All the numbers have to be facing up.



Puzzle retrieved from: "Fractions Jigsaw." : Nrich.maths.org. University of Cambridge, n.d. Web. 21 Mar. 2013. <http://nrich.maths.org/5467>.

Problem Set

1. Solve the following equations.

a) $\frac{8}{15} + \frac{3}{10} =$

b) $\frac{13}{15} - \frac{7}{24} =$

c) $\frac{5}{14} \times \frac{7}{8} =$

d) $\frac{7}{20} \div \frac{5}{7} =$

2. Change all mixed fractions to improper fractions, or vice versa.

a) $\frac{17}{7}$

d) $3\frac{2}{5}$

b) $\frac{37}{8}$

e) $\frac{29}{9}$

c) $2\frac{5}{8}$

f) $4\frac{3}{4}$

3. Circle all fractions equivalent to $\frac{5}{3}$.

a) $\frac{10}{15}$

d) $\frac{15}{9}$

b) $2\frac{1}{3}$

e) $\frac{35}{30}$

c) $1\frac{2}{3}$

f) $\frac{30}{18}$

4. Solve the expression: $(\frac{4}{15} \times \frac{9}{8}) + (\frac{8}{6} \div \frac{5}{12}) - 2\frac{5}{12} + \frac{9}{4}$.

5. Match the ratio with its equivalent fraction.

1. $\frac{8}{16}$

a. $2 : 5$

2. $\frac{8}{12}$

b. $5 : 8$

3. $\frac{15}{24}$

c. $7 : 5$


4. $1\frac{6}{15}$

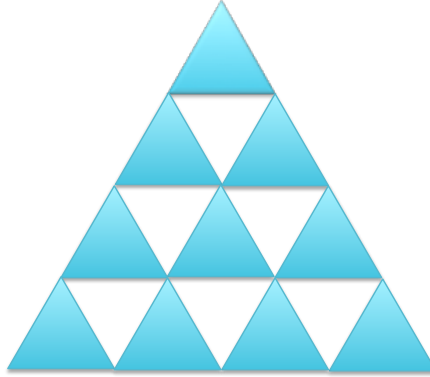
d. $1 : 2$

5. $\frac{32}{80}$

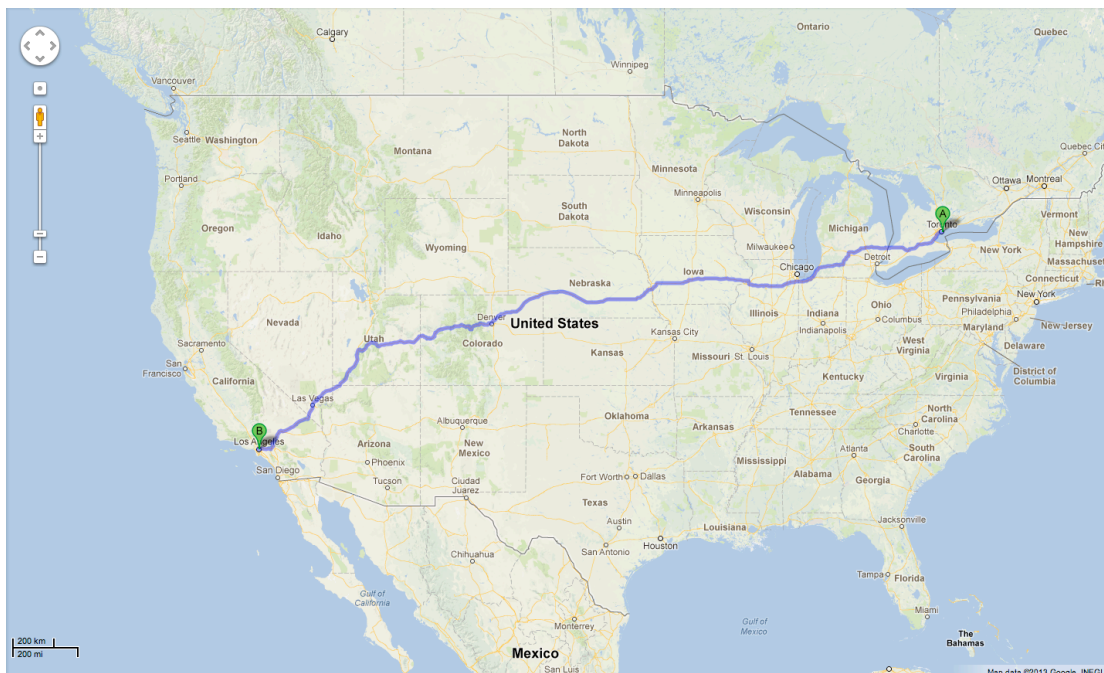
e. $2 : 3$

6. A class has 32 kids. 12 of them are girls and 20 of them are boys. What is the girl to boy ratio in the class?
7. A box is separated into 5 columns and 4 rows. If the first two columns are filled with blue beads, and half of the third column is filled with red beads, what is the red to blue bead ratio? What is the blue bead to box ratio?

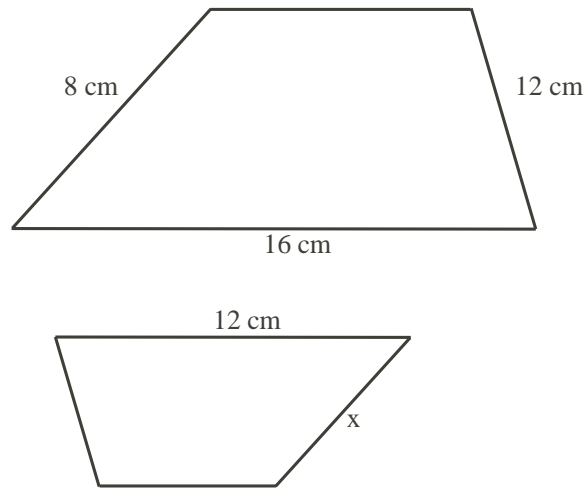
8. How many similar triangles to this equilateral triangle  , are there in the picture below?



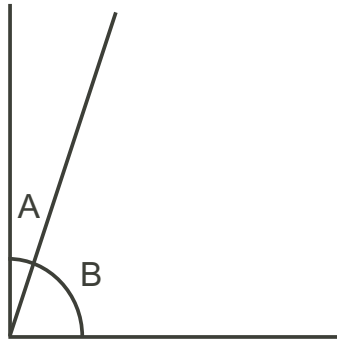
9. If a red rectangle has length 3 units, and a yellow rectangle has length 9 units, what can we say about the ratio of the lengths of the two rectangles?
10. A building is 950 meters tall. A model of the building is made using the ratio 1 : 5,000. How tall is the model in centimeters?
11. A real horse is 1.6 m high. A statue of the horse is 4 m high. What is the ratio of the height of the statue to the height of the horse?
12. What is the distance between Toronto and Los Angeles?



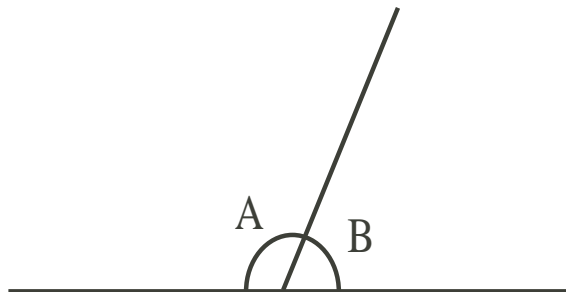
13. The distance between two towns on a map is 6 cm. If the real distance between the two towns is 24 km, what is the scale of the map?
14. The two trapezoids are similar. What is the value of x ?



15. A right angle like below has an angle of 90° . If $\angle A : \angle B = 1 : 4$, what is the size of $\angle B$?



16. A straight line has an angle of 180° . If $\angle A : \angle B = 9 : 6$, what is the size of $\angle A$?



Solutions

1. (a) $\frac{5}{6}$

(b) $\frac{23}{40}$

2. (a) $2\frac{3}{7}$

(b) $4\frac{5}{8}$

(c) $\frac{21}{8}$

(c) $\frac{5}{16}$

(d) $\frac{49}{100}$

(d) $\frac{17}{5}$

(e) $3\frac{2}{9}$

(f) $\frac{19}{4}$

3. c); d); f)

4. $\frac{10}{3}$

5. $1+d$; $2+e$; $3+b$; $4+c$; $5+a$

6. $2 : 5$

7. $1 : 5$; $1 : 2$

8. 27

9. $1 : 3$

10. 19 cm

11. $40 : 16 = 5 : 2$

12. Approx. 4120 km

13. $1 : 400,000$

14. $x = 6$ cm

15. 90° needs to be divided into a ratio of $1 : 4$, so $1 + 4 = 5 \rightarrow \angle B = \frac{4}{5} \times 90^\circ = 72^\circ$

16. 180° needs to be divided into a ratio of $9 : 6$, so $9 + 6 = 15 \rightarrow \angle A = \frac{6}{15} \times 90^\circ = 108^\circ$